AIAA JOURNAL
Vol. 32, No. 5, May 1994

Screech Tones from Free and Ducted Supersonic Jets

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It is well known that screech tones from supersonic jets are generated by a feedback loop. The loop consists of three main components. They are the downstream propagating instability wave, the shock cell structure in the jet plume, and the feedback acoustic waves immediately outside the jet. Evidence will be presented to show that the screech frequency is largely controlled by the characteristics of the feedback acoustic waves. The feedback loop is driven by the instability wave of the jet. Thus the tone intensity and its occurrence are dictated by the characteristics of the instability wave. In this paper the dependence of the instability wave spectrum on the azimuthal mode number (axisymmetric or helical/flapping mode, etc.), the jet-to-ambient gas temperature ratio, and the jet Mach number are studied. The results of this study provide an explanation for the observed screech tone mode switch phenomenon (changing from axisymmetric to helical mode as Mach number increases) and the often-cited experimental observation that tone intensity reduces with increase in jet temperature. For ducted supersonic jets screech tones can also be generated by feedback loops formed by the coupling of normal duct modes to instability waves of the jet. The screech frequencies are dictated by the frequencies of the duct modes. Super resonance, resonance involving very large pressure oscillations, can occur when the feedback loop is powered by the most amplified instability wave. It is proposed that the observed large amplitude pressure fluctuations and tone in the test cells of Arnold Engineering Development Center were generated by super resonance. Estimated super-resonance frequency for a Mach 1.3 axisymmetric jet tested in the facility agrees well with measurement.

I. Introduction

IT is well known that under certain operating conditions imperfectly expanded supersonic jets emit strong discrete tones commonly referred to as screech tones (see Ref. 1 for a recent review of the subject). In the near field the intensity of these tones can be as high as 160 dB. At this sound pressure level the tones can cause structural fatigue and other undesirable vibratory problems. Because of this there is currently a renewed interest in obtaining a better understanding of the screech phenomenon.

Powell[2–4] and Hammitt[5] were the first investigators to study screech tones. Powell investigated the flow and the adjacent acoustic fields of screeching jets optically. Based on the results of his observations he was able to conclude that the screech tones were generated by feedback loops. According to Powell, the feedback loop of each tone consists of downstream propagating flow disturbances initiated at the nozzle lip region. As these disturbances passed through the third or fourth shock cell strong interaction with the oblique shocks took place resulting in the emission of intense acoustic waves. Part of the acoustic waves propagated upstream outside the jet flow. Upon reaching the nozzle lip these acoustic waves triggered the generation of downstream propagating flow disturbances and thus completed the feedback loop.

The feedback loop concept of Powell is basically correct. However, there is a much better understanding of the details of each link of the loop at the present time. Figure 1 shows a schematic diagram of the screech tone feedback loop as it is generally accepted today. An instability wave of the jet is generated by acoustic disturbances near the nozzle exit where the mixing layer is thin and most receptive to excitation. The instability wave grows as it propagates downstream by extracting energy from the mean flow of the jet. At a distance of about four to five shock cells downstream the instability wave, having acquired a large enough amplitude, interacts strongly with the shock cell structure inside the jet plume. This unsteady interaction results in the emission of intense acoustic waves, some of which propagate upstream outside the jet. Upon reaching the nozzle exit the acoustic disturbances excite the shear layer of the jet thus generating a new instability wave. In this way the feedback loop is closed.

The feedback loop consists of three basic components, namely, the feedback acoustic waves, the instability wave, and the shock

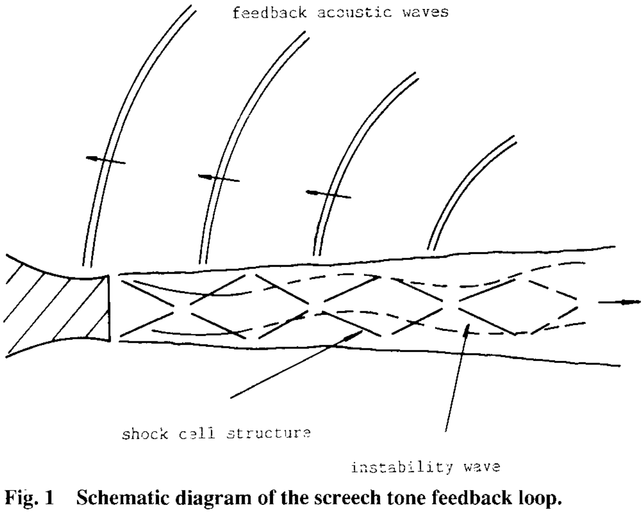

Fig. 1 **Schematic diagram of the screech tone feedback loop.**

Presented as Paper 93-0732 at the AIAA 31st Aerospace Sciences Meeting, Reno, NV, Jan. 11–14, 1993; received March 18, 1993; revision received Nov. 8, 1993; accepted for publication Nov. 16, 1993.

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cell structure. Obviously each of these components plays a different but essential role in the maintenance of the feedback loop. Tam et al.[6] examined the feedback loop and suggested that the feedback acoustic waves were the weakest link of the loop. As in the case of broadband shock-associated noise, the principal direction of radiation of the feedback acoustic waves is related to its frequency. For enough acoustic waves to reach the nozzle lip and maintain the feedback the principal direction of radiation must be in the 180-deg upstream direction. By imposing this upstream radiation condition Tam et al. derived the following screech tone frequency formula

$$\frac{fD_j}{U_j} = 0.67\,(M_j^2 - 1)^{-1/2}\left[1 + 0.7M_j \cdot \left(1 + \frac{\gamma - 1}{2}M_j^2\right)^{-1/2}\left(\frac{T_r}{T_a}\right)^{1/2}\right]^{-1} \tag{1}$$

where f is the screech tone frequency and U_j is the fully expanded jet velocity. M_j is the fully expanded jet Mach number. D_j, the fully expanded jet diameter, is related to the nozzle exit diameter D and nozzle design Mach number M_d by[7]

$$\frac{D_j}{D} = \left[\frac{1 + (1/2)\,(\gamma - 1)\,M_j^2}{1 + (1/2)\,(\gamma - 1)\,M_d^2}\right]^{\frac{(\gamma+1)}{4(\gamma-1)}}\left(\frac{M_d}{M_j}\right)^{1/2} \tag{2}$$

T_r is the total jet temperature and T_a is the ambient gas temperature. They demonstrated by comparing with experiments that this formula could provide reasonably accurate (staging of tones not predicted) prediction of the screech frequency.

Equation (1) is valid for hot as well as for cold jets. Figure 2 shows a comparison of the calculated screech frequencies and the measurements of Rosfjord and Toms[8] at three jet temperatures. As can be seen, there is good agreement at all temperatures. The shift of the screech frequency to higher values due to higher jet temperature is correctly predicted. Recently a sequence of hot jet screech tone experiments were carried out by the present investigators using a 1-in. convergent nozzle. Figure 3 shows comparisons of the measured data and the predictions of Eq. (1) at temperature ratios of 1.0, 2.24, and 2.76. Again there is reasonable agreement (staging of tone frequencies not predicted). Once more Eq. (1) seems to give good agreement at high temperature ratios. Based on the good comparisons with experiments given in Ref. 6 and the

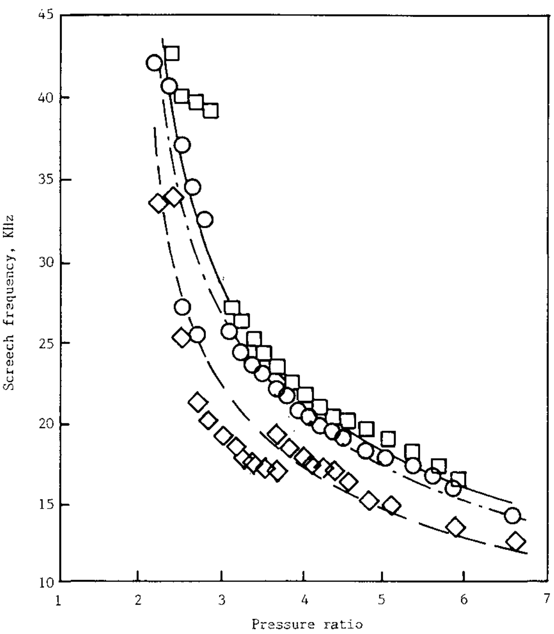

Fig. 2 Dependence of screech tone frequency on jet pressure ratio at different total temperature T_r for convergent nozzle; data from Ref. 8: ◇, T_r = 18°C, ---- Eq. (1); ○, T_r = 323°C, -·-·- Eq. (1); and □, T_r = 529°C, —— Eq. (1).

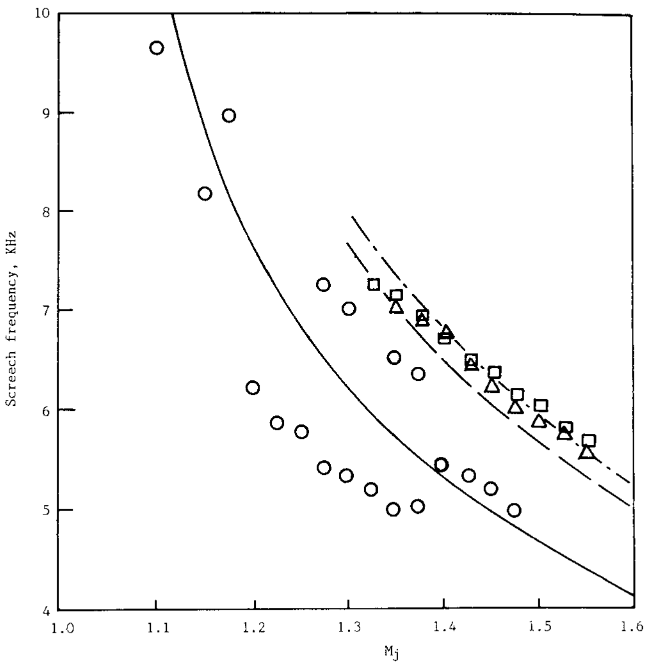

Fig. 3 Dependence of screech tone frequency on jet Mach number at different temperature ratios: ○, T_r/T_a = 1.0, —— Eq. (1); △, T_r/T_a = 2.24, ---- Eq. (1); and □, T_r/T_a = 2.76, -·-·- Eq. (1).

preceding results, one must conclude that the screech frequency is determined largely by the characteristics of the feedback acoustic waves. The instability wave and the shock cell structure, though they may have a deciding influence on the other characteristics of the screech tones, affect the tone frequency indirectly or only in a secondary role.

When a supersonic jet is put inside a duct (referred to as ducted jet) screech tones other than those related to the shock cells can be found.[9,10] It is well known that a duct can support several families of stationary normal acoustic modes. The insertation of a jet inside the duct allows the instability waves of the jet to couple to these duct modes producing feedback resonance. Figure 4 shows a schematic diagram of the feedback loop. Here the acoustic disturbances associated with a normal mode of the duct excite the instability wave of the jet near the nozzle exit. As the instability wave grows in amplitude as it propagates downstream it causes the jet to oscillate. The oscillatory motion of the jet pumps energy back into the normal acoustic mode and closes the feedback loop. The screech frequencies of these ducted jets are nearly equal to the frequencies of the normal duct modes. There is a slight shift due to the entrained flow. This is especially true for screech tones involving the higher order duct modes.

The purpose of the present investigation is to examine the influence of the instability waves of the jet, the second element of the feedback loop, on the characteristics of the screech tones. Both free and ducted jets are considered. As the instability waves of the jet are the source of energy which powers the feedback loops generating the screech tones it is anticipated that they would dictate the intensities and occurrence of the tones. The frequencies are, however, largely controlled by the feedback acoustic waves as was noted before. It is known[11,12] that for cold jets the screech tones are axisymmetric with respect to the jet axis at low supersonic jet Mach number (toroidal mode). At higher jet Mach numbers they switch to the helical/flapping mode. The mode-switching phenomenon is as yet not understood. An explanation of this mode switching will be provided in this paper. Another well-known empirical fact is that screech tone intensity decreases with an increase in jet temperature. Because jet engines are invariably operated at an elevated temperature, screech tones are sometimes regarded by engineers not to be an important component of aircraft noise. The cause of the decrease in tone intensity with increase in temperature

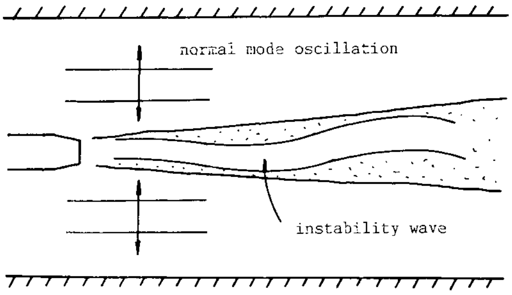

Fig. 4 Schematic diagram of a feedback loop formed inside a duct between an acoustic normal duct mode and a large-scale instability wave of the jet.

is still unknown. Obtaining an understanding of the cause is one of the objectives of the present investigation.

The screech frequencies of ducted jets are controlled primarily by the normal modes of the duct. The intensity, on the other hand, is very much influenced by the characteristics of the instability waves of the jet. It has been found that under special operating conditions a ducted jet may exhibit violent resonant oscillation. This is most undesirable in engine test cell operations.[13] It turns out that the violent jet oscillation, here referred to as super resonance, is closely tied to the behavior of jet instability. An analysis of the phenomenon of super resonance will be given in this work.

II. Amplified Instability Wave Spectrum

A supersonic jet is unstable only over a finite range of frequencies. As the instability waves of the jet are the energy source of screech tones it follows that the screech frequencies must lie within the unstable frequency band. Linear instability wave theory indicates that the growth rate of a wave depends on its frequency, the shear layer thickness or the mean velocity profile, the jet Mach number and temperature, and the azimuthal mode number. Thus, waves at different frequencies will be amplified differently. The wave with the largest amplification or total growth would attain the highest amplitude, hence it would most likely generate screech tone of the highest intensity. In this section the characteristics of the amplified instability wave spectrum is investigated.

Linear instability wave theory is now a relatively well-established subject. The mathematical and computational procedure needed to determine the spatial growth rates of these waves are fully documented in the literature (e.g., see Refs. 14 and 15). Recently, in a study of turbulent mixing noise from supersonic jets,[16] this computational procedure was used to calculate the total spatial growth of the instability wave spectrum. Here the same method will be used to compute the total amplification of instability waves for jet parameters relevant to screech tones.

In Ref. 16 it was pointed out that except at very low frequencies, instability waves generally attain their maximum amplitude in the core region of the supersonic jet. For convenience, a cylindrical coordinate system (r, θ, x) centered at the nozzle exit with the x axis pointing in the direction of the flow will be used. In the core region of the jet the mean velocity profile is adequately represented by a two parameter function,

$$\frac{U(r,x)}{U_j} = \begin{cases} 1, & r \le h \\ \exp\{-ln2\,[(r-h)/b]^2\} & r > h \end{cases} \tag{3}$$

where U_j is the fully expanded jet velocity, h is the core radius, and b is the half-width of the mixing layer. Both h and b are functions of the downstream coordinate x. They are related by the requirement of conservation of momentum flux, i.e.,

$$\int_0^\infty \rho U^2 r\,\mathrm{d}r = \frac{1}{2}\rho_j U_j^2 R^2 \tag{4}$$

where ρ is the gas density, and ρ_j and R are the fully expanded density and radius of the jet, respectively. The pressure across the jet is taken to be constant by the boundary-layer approximation. The density ρ is related to the mean velocity U through the Crocco's relation. The explicit relation is

$$\frac{\rho_j}{\rho} = \left(1+\frac{\gamma-1}{2}M_j^2\right)\left[\frac{T_a}{T_r}+\left(1-\frac{T_a}{T_r}\right)\frac{U}{U_j}\right] - \frac{\gamma-1}{2}M_j^2\left(\frac{U}{U_j}\right)^2 \tag{5}$$

On combining Eqs. (3), (4), and (5) it is clear that the mean profile of a supersonic jet may be characterized by the single parameter b.

For jet flows with velocity profiles given typically by that of Eq. (3) the instability is inflexional. Such instability is called the Kelvin-Helmholtz instability. Inflexional instability requires no viscosity and has a large spatial growth rate. The linear instability wave solution under the locally parallel flow approximation is governed by the Rayleigh equation. With axisymmetric mean flow the solution may be expressed in the form $p(r, \theta, x, t) = \hat{p}(r)\exp[i(kx + n\theta - \omega t]$ and in similar form for the other variables. Here ω is the angular frequency, n the azimuthal wave number ($n = 0, 1, 2, \dots$), and k the axial wave number and the eigenvalue of the problem. The negative value of the imaginary part of k or $-k_i$ is the local growth rate of the instability wave. As the wave propagates downstream it sees a continuous change in the mean velocity and density profiles. Its spatial growth rate, therefore, changes as well. For a wave with a fixed frequency f or Strouhal number, $S = fD_j/U_j$, and azimuthal mode number n in a supersonic jet of Mach number M_j and temperature ratio T_r/T_a, the total amplification $A(S, n, M_j, T_r/T_a)$ is given by

$$A(S, n, M_j, T_r/T_a) = \exp\left[-\int_0^{x_c} k_i(x)\,\mathrm{d}x\right] \tag{6}$$

where x_c is the location at which the wave reaches its maximum amplitude or the local growth rate is zero. Extensive experimental data indicate that b varies nearly linearly with x. That is, $\mathrm{d}b/\mathrm{d}x = \sigma$ is a constant. By means of the linear relationship between b and x the total growth integral on the right side of Eq. (6) may be rewritten in the dimensionless form

$$I = -\int_{b_0/R}^{b_c/R} k_i(b/R)R\,\mathrm{d}(b/R) \tag{7}$$

where b_0 is the half-width of the mixing layers at the nozzle exit and $b_c = b(x_c)$.

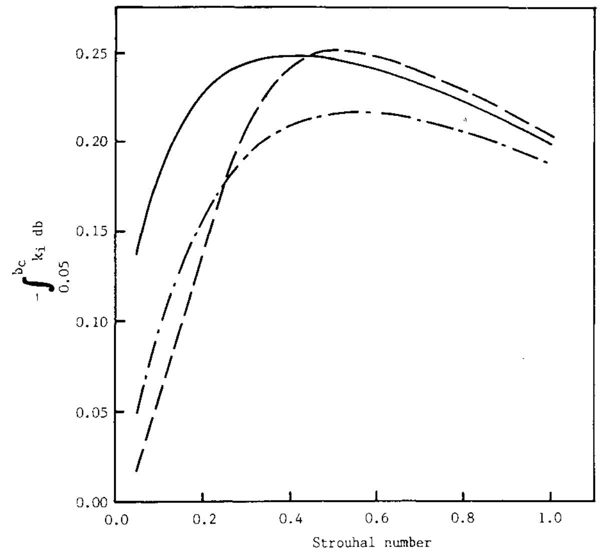

Fig. 5 Total growth integral of the instability waves of a Mach 1.1 (cold) supersonic jet as a function of Strouhal number (fD_j/U_j): ---- $n = 0$; —— $n = 1$; and —·—·— $n = 2$ mode.

Figure 5 shows the values of the total growth integral of the lowest three azimuthal modes, $n = 0, 1, 2$, as functions of Strouhal number for a Mach 1.1 cold jet ($T_r = T_a$). This figure shows that the axisymmetric mode ($n = 0$) has a slightly higher total growth than the other modes. The maximum amplification occurs at Strouhal number 0.5. The helical mode ($n = 1$) has a slightly smaller total amplification which peaks at a Strouhal number of 0.4. It is found, however, that the total amplification of the axisymmetric mode becomes smaller relative to the helical mode as the jet Mach number increases. At jet Mach number 1.4 the helical mode has the highest amplification of all the modes as can easily be seen in Fig. 6. This means that at low supersonic Mach numbers the axisymmetric mode dominates. The dominance gradually diminishes as the Mach number increases. At about $M_j = 1.3$ the helical mode becomes the dominant mode. Figure 7 shows the most amplified instability wave Strouhal number for the axisymmetric and helical

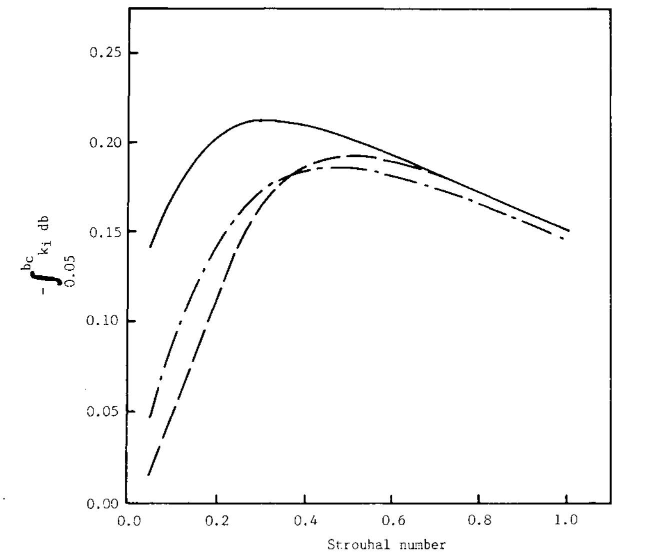

Fig. 6 Total growth integral of the instability waves of a Mach 1.4 (cold) supersonic jet as a function of Strouhal number: ---- $n = 0$; —— $n = 1$; and –·–·– $n = 2$ mode.

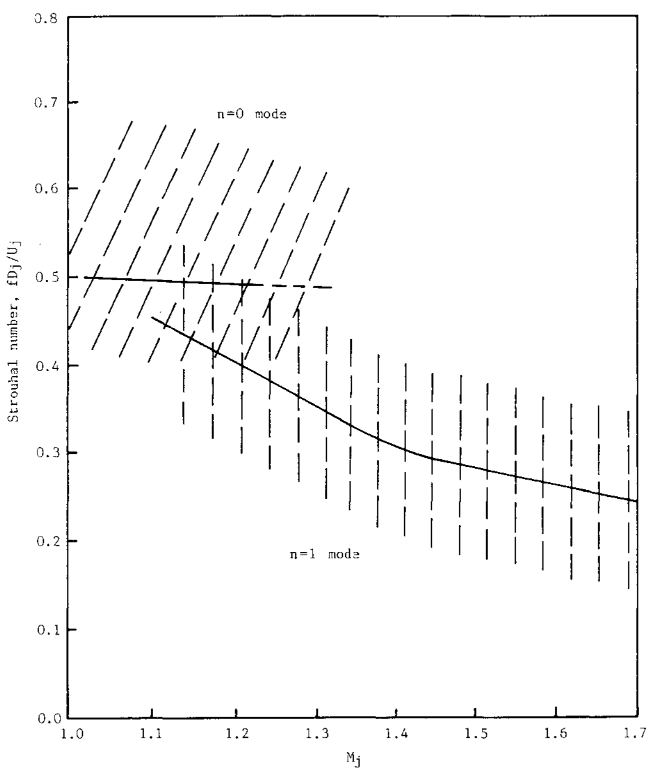

Fig. 7 Range of the most amplified axisymmetric and helical instability wave modes of cold supersonic jets.

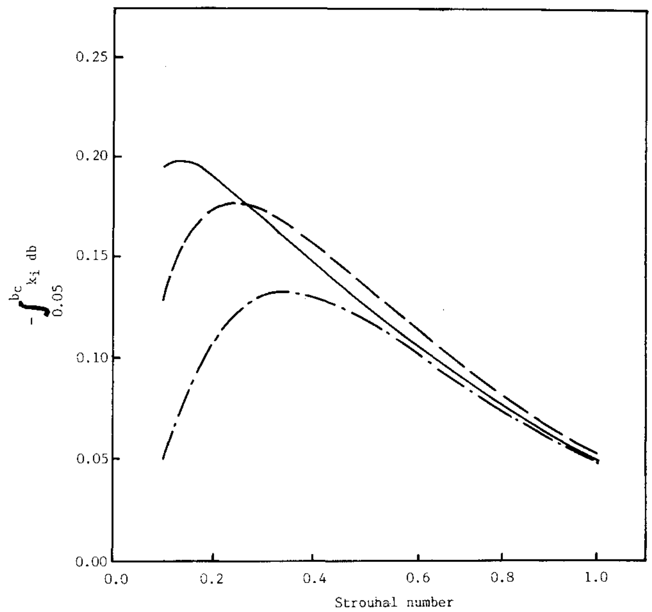

Fig. 8 Total growth integral of the instability waves of a Mach 1.3 jet at $T_j/T_a = 2.0$ vs Strouhal number: ---- $n = 0$; —— $n = 1$; and –·–·– $n = 2$ mode.

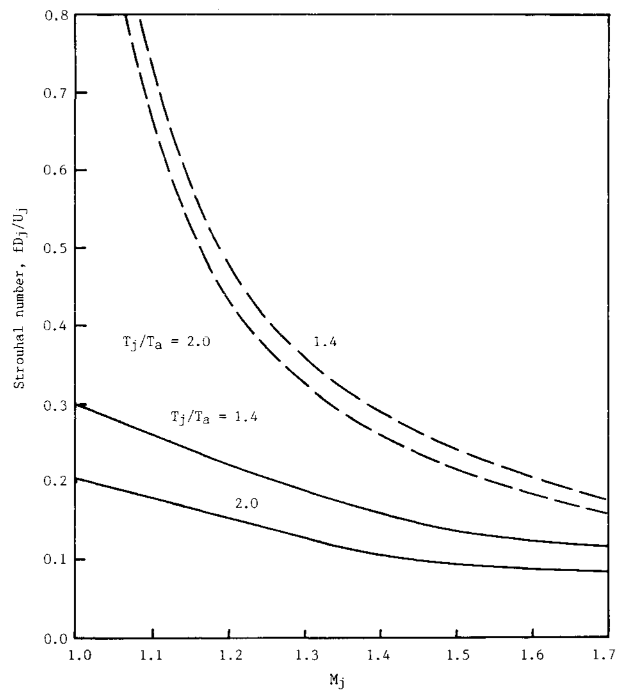

Fig. 9 Dependence of the most amplified instability wave ($n = 1$) curve, —— and the screech tone frequency curve, Eq. (1), ---- on jet Mach number.

mode instability waves as functions of the jet Mach number ($T_r = T_a$). Obviously the axisymmetric mode wave peaks at a higher frequency and is an important wave mode only up to a Mach number of 1.3. The helical instability wave peaks at a lower frequency. It is the most amplified wave for $M_j > 1.3$. The shaded regions in Fig. 7 represent the parameter space in which the unstable waves have significant total growth and hence are most likely to be observed.

The growth rates of the Kelvin-Helmholtz instability waves are strongly affected by the jet temperature. As the jet-to-ambient temperature ratio increases the total growth of the axisymmetric mode instability waves become smaller and smaller when compared with the helical mode waves. Thus they become less and less important. Figure 8 shows the total growth integrals of the $n = 0$, 1, and 2

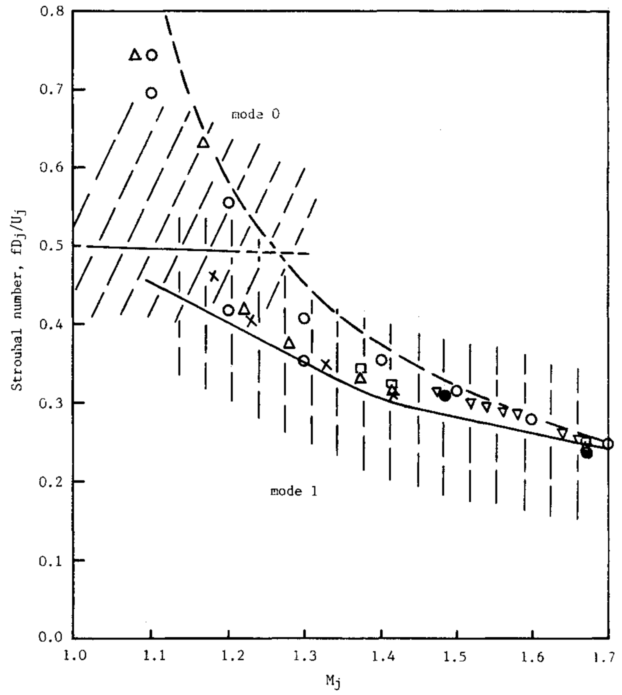

Fig. 10 Comparison of measured screech tone frequencies, Eq. (1) and the range of the most amplified instability wave of cold supersonic jets: ○ present data; Δ Mach 1 nozzle; □ Mach 1.5 nozzle; • Mach 2 nozzle, Ref. 17; x Ref. 18; ∇ Ref. 12; and ---- Eq. (1).

modes as functions of Strouhal number for a Mach 1.3 hot jet at $T_j/T_a = 2.0$ (T_j is the fully expanded jet temperature). Clearly the helical instability wave mode ($n = 1$) is dominant. The most amplified instability wave now has a Strouhal number of 0.13. This is substantially lower than that of a cold jet. In general, the Strouhal number of the most amplified instability wave decreases with an increase in T_j/T_a. In addition, it decreases with an increase in jet Mach number. For reference purposes a plot of the Strouhal numbers of the most amplified instability wave ($n = 1$) given as functions of Mach numbers at $T_j/T_a = 1.4$ and 2.0, is presented in Fig. 9.

III. Screech Tones and Instability Waves

Data obtained by Norum[11] and Seiner et al.[12] indicate that screech tones from cold jets operating at low supersonic Mach number are axisymmetric with respect to the jet axis. However, at higher jet Mach numbers they have a helical or flapping configuration. Now an explanation to this as-of-yet-unresolved mode-switching phenomenon can be given in terms of the instability wave characteristics. Because the screech tones are powered by the instability waves the naturally occurring tones must be associated with the highly amplified band of instability waves. The only constraint is that the frequencies must be close to that given by Eq. (1) (representing the constraint imposed by the feedback acoustic waves). In Sec. II it was found that at low supersonic Mach numbers the axisymmetric instability wave mode is dominant. Thus one would expect the screech tones be axisymmetric with Strouhal numbers lying in the upper-left shaded region of Fig. 7. At higher jet Mach numbers the helical/flapping mode instability is dominant. It follows that the tones would also have helical or flapping configuration. Figure 10 is a replica of Fig. 7 with the screech tone Strouhal number curve of Eq. (1) added. It is noted that the most amplified instability wave curve and the screech frequency curve are very close to each other so that the frequency constraint is easily satisfied as long as the tones lie in the shaded region of the figure. Also shown in this figure are screech tone frequency data measured by various investigators.[12,17,18] The tone data below $M_j = 1.2$ are associated with the axisymmetric instability mode. The data above $M_j = 1.3$ are all associated with the helical mode jet instability. Therefore the switchover of screech tones from the axisymmetric to the helical mode at around $M_j = 1.2$–1.3 is simply the result of the change in the dominance of the instability waves of the jet.

In Sec. II it was pointed out that as the jet temperature increases the Strouhal number of the band of the most amplified instability waves decreases. Although the jet velocity increases as $T_j^{1/2}$ the decrease in Strouhal number is faster than $T_j^{-1/2}$. The net effect is that there is an actual decrease in the frequency of the most amplified instability waves. Measured screech tone data shown in Figs. 2 and 3, and the prediction of Eq. (1), on the other hand, indicate that the screech tone frequencies increase with jet temperature. Hence, for hot jets the screech feedback loops are not driven by the band of the most amplified instability waves. Not surprisingly the tone amplitudes are reduced. Figure 9 shows the Strouhal number of the most amplified instability waves at jet-to-ambient temperature ratios of 1.4 and 2.0. Also shown are the corresponding screech tone Strouhal number curves. On comparing these results with the cold jet curves in Fig. 10 it is clear that the screech tone curves and the jet (most amplified) instability wave curves move apart as jet temperature increases. At low supersonic Mach numbers the two curves are so far apart that screech tones may not be generated at all, as is the case in the data set of Fig. 3. In summary, the observation that hot jets do not emit screech tones or only emit tones at greatly reduced intensity is the direct result of a frequency mismatch between screech tones and the band of the most amplified instability waves of the jet.

IV. Super Resonance

In a recent paper, Jones and Lazalier[13] described certain violent acoustic resonances encountered in the Arnold Engineering Development Center (AEDC) high-altitude turbine engine test facility. Figure 11 is a schematic diagram of the baseline configuration of the test cell. They reported that when an axisymmetric jet from a convergent nozzle two feet in diameter, at a total temperature of approximately 1800°R and a Mach number of 1.3 was being tested, the jet underwent violent oscillation and a discrete tone at 140 Hz was emitted. The cause of the resonance was unknown. However, they were able to suppress the resonance by conventional engineering methods.

The test cell resonance is evidently related to the ducted jet feedback resonance illustrated schematically in Fig. 4. The normal modes, in this case, are the acoustic modes of the cylindrical/conical duct piece of the facility as is shown in Fig. 11. The geometry of the cylindrical/conical duct piece is very similar to that of a circular duct of diameter D and length L with one open and one closed end. The normal mode frequencies f_{lmn} of such a finite-length duct are given by

$$f_{lmn} = \left[\sigma_{nm}^2 + \left(l + \frac{1}{2}\right)^2\left(\frac{\pi D}{2L}\right)^2\right]^{1/2} \frac{a_0}{\pi D}$$

$$l, n = 0, 1, 2, \ldots \qquad (8)$$

$$m = 1, 2, 3, \ldots$$

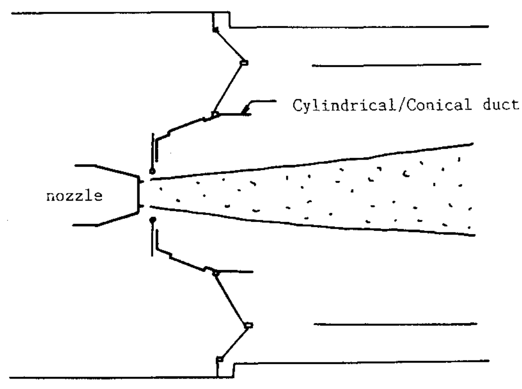

Fig. 11 Baseline configuration of the test cells at AEDC.

where l, n, and m are the longitudinal, azimuthal, and radial mode numbers, respectively. Here, a_0 is the sound speed inside the duct. σ_{nm} is the mth root of J_n' (prime denotes derivative); i.e., $J_n'(\sigma_{nm}) = 0$. J_n is the Bessel function of order n. The presence of a jet inside a finite-length duct invariably modifies the acoustic normal mode frequencies. An analysis of the presence of a vortex sheet jet inside a duct has been carried out. Numerical results, however, reveal that the jet alters the low-order duct mode frequencies by only a negligible amount. Therefore, as a first estimate, Eq. (8) is used. The diameter of the cylindrical/conical duct piece is not exactly constant. It varies from 10.75 ft at the open end to 8.5 ft at the closed end. The average diameter is 9.63 ft, which is used in the following computation. The length of the duct piece is 8.08 ft. Jones and Lazalier estimated that the ambient gas surrounding the jet has a temperature of about 250°F or 710°R so that the sound speed a_0 is 1300 ft/s. On noting that $\sigma_{11} = 1.841$ ($n = m = 1$), it is easy to find from Eq. (8) that the normal mode frequency of the (1, 1, 1) mode; i.e., f_{111}, is equal to 144 Hz. This is very close to the observed tone frequency of 140 Hz. The good agreement suggests that the test cell resonance is primarily related to the $n = \pm 1$ azimuthal (helical or flapping) jet instability wave mode.

As the observed test cell resonance is associated with extremely high tone intensity and large amplitude jet oscillations, it is probably not just a case of simple resonance. From an energy standpoint the strong oscillation has to be driven by a large amplitude instability wave of the jet (the only large source of energy available). As a working hypothesis on the mechanism responsible for the test cell screech phenomenon, it is proposed that the oscillation was driven by a feedback loop consisting of the (1, 1, 1) mode of the inlet cylindrical/conical duct and the instability wave of the jet. Because the oscillation was reported to be very violent the instability wave involved must be the most amplified wave, a situation hereby referred to as super resonance. To demonstrate that the test cell tone and pressure fluctuations are indeed generated by super resonance we must first determine the frequency of the most amplified instability wave of the jet at the test condition.

As was mentioned earlier, Jones and Lazalier estimated that the ambient gas surrounding the jet has a temperature of about 250°F or 710°R. By taking γ to be 1.35 and the jet total temperature to be 1800°R, the jet temperature T_j is easily found to be 1389°R. Thus T_j/T_a is equal to 1.96, say 2.0. For the jet in question ($M_d = 1.0$) the fully expanded jet diameter may be calculated by Eq. (2) giving $D_j = 2.066$ ft. At a jet temperature of 1389°R the velocity of a Mach 1.3 jet U_j is 2337 ft/s. Now Fig. 8 applies to this case. The Strouhal number fD_j/U_j of the most amplified instability wave is 0.13. The corresponding frequency is equal to $0.13U_j/D_j = 147$ Hz. This is very close to the measured resonance frequency of 140 Hz. Therefore, there is good reason to believe that super resonance was really responsible for the observed tone and test cell pressure oscillations at AEDC.

V. Summary and Discussion

Screech tones from free or ducted supersonic jets are generated by a feedback loop. The screech frequencies are largely dictated by the feedback acoustic waves in the case of freejets and by the normal modes of the duct in the case of ducted jets. The feedback loop is driven by the instability waves of the jet. Thus the tone intensity is governed primarily by the characteristics of the instability wave.

Instability waves of a supersonic jet are limited to a finite frequency band. The total growth of the instability waves can be estimated by linear instability wave theory. For a given jet operating condition, one family of azimuthal instability wave mode is generally dominant, i.e., most amplified. It is shown in this paper that the switch of dominant screech mode (axisymmetric to helical/flapping) as Mach number increases is due to the switch in dominance of the corresponding mode of instability waves. Further, the decrease in screech tone intensity with jet temperature occurs because the frequencies of the band of the most amplified instability waves decrease with jet temperature while the screech frequencies increase. The result is that the less amplified waves are involved in driving the screech tone feedback loop. It follows that the tone intensities are reduced.

In ducted jets super resonance can occur when the most amplified instability wave and a normal mode of the duct form a feedback loop. The intensity of the tone generated and the amplitude of pressure fluctuation are expected to be very large. It is suggested that the large pressure fluctuations observed at the AEDC test cell facility were driven by super resonance. The calculated most amplified instability wave frequency agrees well with the measured resonance frequency.

So far the possibility of predicting screech tone intensity has not been considered. To do so the role of the shock cell structure in the screech tone feedback loop must first be clarified and studied. This is desirable but is beyond the scope of the present investigation.

Acknowledgments

This work was supported by Arnold Engineering Development Center and NASA Langley Research Center under Contract NAS1-19061.

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